

# Measuring $|V_{ub}|$ with $B \rightarrow D_s^+ X_u$ transitions

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## Abstract

We propose the determination of the CKM matrix element  $|V_{ub}|$  by the measurement of the spectrum of  $B \rightarrow D_s^+ X_u$ , dominated by the spectator quark model mechanism  $\bar{b} \rightarrow D_s^{(*)+} \bar{u}$ . The interest of considering  $B \rightarrow D_s^+ X_u$  versus the semileptonic decay is that more than 50 % of the spectrum for  $B \rightarrow D_s^+ X_u$  occurs above the kinematical limit for  $B \rightarrow D_s^+ X_c$ , while most of the spectrum  $B \rightarrow l \nu X_u$  occurs below the  $B \rightarrow l \nu X_c$  one. Furthermore, the measure of the hadronic mass  $M_X$  is easier in the presence of an identified  $D_s$  than when a  $\nu$  has been produced. As a consistency check, we point out that the rate  $\bar{b} \rightarrow D_s^{(*)+} \bar{c}$  (including QCD corrections that we present elsewhere) is consistent with the measured  $BR(B \rightarrow D_s^\pm X)$ . Although the hadronic complications may be more severe in the mode that we propose than in the semileptonic inclusive decay, the end of the spectrum in  $B \rightarrow l \nu X_u$  is not well understood on theoretical grounds. We argue that, in our case, the excited  $D_s^{**}$ , decaying into  $DK$ , do not contribute and, if there is tagging of the  $B$  meson, the other mechanisms to produce a  $D_s$  of the right sign are presumably small, of  $O(10^{-2})$  relative to the spectator amplitude, or can be controlled by kinematical cuts. In the absence of tagging, other hadronic backgrounds deserve careful study. We present a feasibility study with the BaBar detector.

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# 1 Introduction

The determination of the strength of the transition between  $b$  and  $u$  quarks is a very important goal for understanding the sector of the theory involving flavor mixing. Indeed, the value of the element  $|V_{ub}|$  in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [1] is a key ingredient which is used to determine the unitarity triangle and thus test the consistency of the Standard Model in the sector responsible for CP violation. It is also one of the most difficult measurements in  $B$  physics, in particular due to the large and model dependent theoretical uncertainties. The methods which have been used so far to extract  $|V_{ub}|$  involve semileptonic  $B$  decays. The first method uses the inclusive lepton spectrum above the kinematical limit for  $b \rightarrow c$  transitions while the second technique requires the exclusive reconstruction of  $B \rightarrow \pi l \nu$  or  $\rho l \nu$ . The errors in the first case are due to the fact that only a tiny fraction of the lepton energy spectrum from  $b \rightarrow u l \nu$  is observed, that parton model evaluation is questionable in this kinematical region and that a large model dependent extrapolation is necessary to extract the total rate. An improvement based on studying the hadronic mass spectrum increases the signal but is not free of problems related to the  $b \rightarrow c$  background [2]. In the second case, the uncertainties are mainly due to the limited statistics and the theoretical uncertainty in the form factors for the  $B \rightarrow \pi$  and  $B \rightarrow \rho$  transitions.

We would like in the following to propose a new approach to measure  $|V_{ub}|$  which involves inclusive  $B \rightarrow D_s^+$  transitions where we make use as much as possible of experimentally measured parameters in order to reduce the uncertainties. In these decays the  $D_s$  meson is essentially produced via the virtual  $W$  emitted by the  $b$

quark (see figure 1). We shall discuss later the other possibilities to produce a  $D_s$  meson and make a preliminary survey of the backgrounds and hadronic uncertainties of our method to measure  $|V_{ub}|$ . The  $b \rightarrow u$  transitions are identified by requiring the momentum of the  $D_s$  meson to be in the range above the kinematical limit for the decay  $B \rightarrow D_s^+ \bar{D}$  (i.e.  $\sim 1.82$  GeV in the B meson center of mass) and up to 2.27 GeV corresponding to the transition  $B \rightarrow D_s^+ \pi$ . It is very important to note here that in contrast to the inclusive semileptonic case *this range includes the majority of the  $\bar{b} \rightarrow D_s^+ \bar{u}$  transitions* and therefore a smaller extrapolation is needed to obtain the total rate. Of course, a drawback of this new method is that, since it concerns purely hadronic transitions, it is subject to other hadronic uncertainties than the semileptonic end spectrum  $B \rightarrow l \nu X_u$ . After calculating the inclusive rate for  $B \rightarrow D_s^+ X_q$  we discuss how  $|V_{ub}|$  is extracted and then enumerate and try to estimate the uncertainties in section 3. Various sources of background are studied and rejection methods are proposed in section 4 for tagged events and in section 5 for untagged events. Finally, in section 6 we present a feasibility study for the BaBar detector, and in section 7 we conclude.

When this paper was finished, we noticed that other methods to measure  $V_{ub}$  have been proposed using channels that involve also the  $(\bar{s}c)(\bar{u}b)$  weak coupling. Namely, the totally *inclusive*  $B$  decays through  $b \rightarrow \bar{c}su$  has been proposed [3] or rare exclusive decays of the type  $B^+ \rightarrow D_s^+ \gamma$  [4]. However, although the weak coupling is the same, these methods do not overlap with the proposition of our paper to measure  $|V_{ub}|$ .

## 2 The $B \rightarrow D_s^+ X_q$ rate

The inclusive decay rate of a B meson decaying into a  $D_s^+$  meson is obtained using the spectator quark model by writing

$$\Gamma(B \rightarrow D_s^+ X_q) \simeq \Gamma(\bar{b} \rightarrow D_s^+ \bar{q}) + \Gamma(\bar{b} \rightarrow D_s^{*+} \bar{q}) \quad (1)$$

where  $q$  is the outgoing quark as shown in figure 1 (other diagrams exists and will be discussed later). One should note that decays to the lowest P wave  $D_s^{**}$  states do not lead to  $D_s$  mesons since their main decays are  $D_s^{**} \rightarrow D^{(*)} K$ .

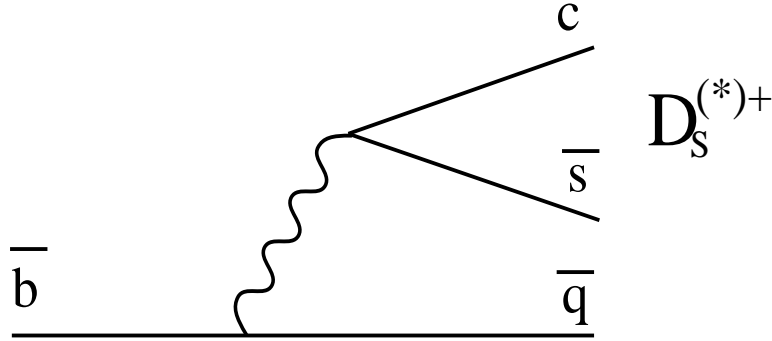


Figure 1: Spectator diagram for the decay  $\bar{b} \rightarrow D_s^{(*)+} \bar{q}$ .

Extending the standard vacuum insertion approximation, successful in exclusive decays, the effective matrix element used for the weak decay  $\bar{b} \rightarrow D_s^{(*)+} \bar{q}$  reads

$$\langle D_s^{(*)+} \bar{q} | \mathcal{H}_{eff} | \bar{b} \rangle = \frac{G_F}{\sqrt{2}} a_1 V_{qb}^* V_{cs} \langle D_s^{(*)+} | A^\mu(V^\mu) | 0 \rangle \langle \bar{q} | J_{\mu q} | \bar{b} \rangle \quad (2)$$

where  $G_F$  is the Fermi constant,  $V_{ij}$  are the Cabibbo-Kobayashi-Maskawa matrix elements and

$$a_1 = c_1 + \frac{c_2}{N_c} \quad (3)$$

is a combination of short distance QCD factors, and the current  $J_{\mu q}$  reads :

$$J_{\mu q} = \bar{q}\gamma_\mu(1 - \gamma_5)b \quad . \quad (4)$$

We have, for the emission of a pseudoscalar :

$$\langle D_s^+ | A^\mu | 0 \rangle = -i f_P p_P^\mu \quad (5)$$

and for the emission of a vector meson

$$\langle D_s^{*+} | V^\mu | 0 \rangle = m_V f_V \epsilon_V^{*\mu} \quad (6)$$

Here  $\epsilon^*$  is the polarisation quadrivector of the meson. In Eq. 3, the Wilson coefficients are [5]

$$c_1 = \frac{c_+ + c_-}{2} \quad \text{and} \quad c_2 = \frac{c_+ - c_-}{2} \quad , \quad c_\pm = \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_W)} \right]^{d_\pm} \quad (7)$$

where  $d_+ = -6/23$  and  $d_- = 12/23$ . In writing Eq. 2, factorization has been assumed. This assumption is justified since the diagram involved here (Fig. 1) is the spectator diagram with external emission of the W. Indeed no internal emission diagram nor penguin diagrams exist. Factorization is so far consistent with the experimental data in exclusive decays where only Fig. 1 type diagrams are involved and the parameter  $|a_1| = 1.00 \pm 0.06$  has been extracted using a combined fit of several measured modes. On the other hand, factorization, up to calculable corrections, has been proved recently in the  $m_b \rightarrow \infty$  limit for  $B \rightarrow \pi\pi$  [6]. However one should be aware that it has been shown [7] that duality between the parton model and the sum over all exclusive channels with the factorization assumption may in general have corrections at the  $1/N_c$  level. Discussions on this point can be found in refs. [8].

The width of the inclusive  $\bar{b} \rightarrow D_s^+ \bar{q}$  is calculated easily at the tree level by evalu-

ating the diagram in figure 1. One finds:

$$\Gamma^{(0)}(\bar{b} \rightarrow D_s^+ \bar{q}) = \frac{G_F^2}{8\pi} |V_{qb}^* V_{cs}|^2 f_{D_s}^2 \frac{(m_b^2 - m_q^2)^2}{m_b^2} \left( 1 - \frac{m_{D_s}^2 (m_b^2 + m_q^2)}{(m_b^2 - m_q^2)^2} \right) p_{D_s} a_1^2 \quad (8)$$

where  $p_{D_s} = \sqrt{[m_b^2 - (m_{D_s} + m_q)^2][m_b^2 - (m_{D_s} - m_q)^2]}/2m_b$  is the momentum of the outgoing  $D_s$  meson in the  $b$  rest frame,  $G_F$  is the Fermi constant and  $f_{D_s}$  is the  $D_s$  decay constant. The notation  $\Gamma^{(0)}$  is used for the width without including the radiative corrections. A similar formula is obtained for  $\Gamma^{(0)}(\bar{b} \rightarrow D_s^{*+}(\lambda = 0) \bar{q})$  where the  $D_s^{*+}$  is longitudinally polarized by replacing in Eq. 8.  $f_{D_s}$  with  $f_{D_s^*}$ ,  $m_{D_s}$  with  $m_{D_s^*}$ . For the transverse polarization ( $\lambda = \pm 1$ ) we find

$$\Gamma^{(0)}(\bar{b} \rightarrow D_s^{*+}(\lambda = \pm 1) \bar{q}) = \frac{G_F^2}{4\pi} |V_{qb}^* V_{cs}|^2 f_{D_s^*}^2 m_{D_s^*}^2 \frac{m_b^2 + m_q^2}{m_b^2} \left( 1 - \frac{m_{D_s^*}^2}{m_b^2 + m_q^2} \right) p_{D_s^*} a_1^2 \quad (9)$$

It is interesting to note that neglecting  $m_{D_s^*}^2$  compared to  $m_b^2$  and for  $m_q^2 \ll m_b^2$ , one obtains

$$\frac{\Gamma_T}{\Gamma_L} = \frac{\Gamma^{(0)}(\bar{b} \rightarrow D_s^{*+}(\lambda = \pm 1) \bar{q})}{\Gamma^{(0)}(\bar{b} \rightarrow D_s^{*+}(\lambda = 0) \bar{q})} \simeq \frac{2m_{D_s^*}^2}{m_b^2 - 4m_q^2} \quad (10)$$

and therefore transverse polarizations are suppressed. As an illustration, Table 1

	$\bar{b} \rightarrow D_s^{*+} \bar{c}$	$\bar{b} \rightarrow D_s^{*+} \bar{u}$
$\Gamma_T/\Gamma_L$	$\sim 1/2$	$\sim 1/3$

Table 1: Fraction of tranverse to longitudinal polarized  $D_s^*$  mesons in inclusive decays.

shows the expected order of magnitude of the ratio  $\Gamma_T/\Gamma_L$  for  $\bar{b} \rightarrow D_s^{*+} \bar{c}$  and  $\bar{b} \rightarrow D_s^{*+} \bar{u}$  transitions. Experimental verifications of table 1 would be useful and would give further confidence in the method proposed here. Adding both longitudinal and

transverse polarizations, one has

$$\Gamma^{(0)}(\bar{b} \rightarrow D_s^{*+} \bar{q}) = \frac{G_F^2}{8\pi} |V_{qb}^* V_{cs}|^2 f_{D_s^*}^2 \frac{(m_b^2 - m_q^2)^2}{m_b^2} \left( 1 + \frac{m_{D_s^*}^2 (m_b^2 + m_q^2 - 2m_{D_s^*}^2)}{(m_b^2 - m_q^2)^2} \right) p_{D_s^*} a_1^2 \quad (11)$$

From (3) and (7) it can be seen that the *short distance* QCD factor  $a_1 = 1 + O(\alpha_s^2)$ , i.e. the correction to the tree rate is of second order in  $\alpha_s$ . We have computed elsewhere [9] the radiative corrections to  $b \rightarrow D_s^{(*)-} u$  at order  $\alpha_s$ , that involve vertex, self-energy and Bremsstrahlung diagrams. These radiative corrections are evaluated at the order  $\alpha_s$  in the same way than for the semileptonic decays [10, 11], i.e. on the lower quark legs in Fig. 1. This is because the  $D_s^{(*)}$  is a color singlet. We have obtained, within the on-shell renormalization scheme [9] :

$$\Gamma(\bar{b} \rightarrow D_s^{(*)+} \bar{q}) = \Gamma^{(0)}(\bar{b} \rightarrow D_s^{(*)+} \bar{q}) \left[ 1 + \frac{4}{3} \frac{\alpha_s}{\pi} \eta^{(*)}(\xi_{D_s^{(*)}}, r_q) \right] \quad (12)$$

where  $\xi = \frac{q^2}{m_b^2}$  and  $r_q = \frac{m_q}{m_b}$ , with  $q^2 = m_{D_s}^2$  or  $m_{D_s^*}^2$ . As shown in [9], in the limit  $\xi \rightarrow 0$ ,  $r_q \rightarrow 0$  one finds

$$\eta(0, 0) = \eta^*(0, 0) = \frac{5}{4} - \frac{\pi^2}{3} \quad . \quad (13)$$

The functions  $\eta^{(*)}(\xi, r)$  are slowly varying with  $r$  and  $\xi$ .

In order to derive the expected branching fractions, the following numerical values are used for the pole quark masses, extracted from an analysis of semi-leptonic  $B$  decays at first order, to be coherent with the present first order calculation (see ref. [9] for a discussion on the choice of these parameters) :

$$m_b = 4.85 \text{ GeV}/c^2, \quad m_c = 1.45 \text{ GeV}/c^2 \quad (14)$$

and we take  $m_u \cong 0$  and the decay constants

$$f_{D_s} = 230 \text{ MeV} \quad , \quad f_{D_s^*} = 280 \text{ MeV} \quad (15)$$

With these values, and  $\alpha_s(m_b) = 0.2$ , the radiative corrections take the following values (the mass dependence is discussed in [9]) for  $q = c$  :

$$\begin{aligned} \frac{4}{3} \frac{\alpha_s}{\pi} \eta(\xi_{D_s}, r_c) &= -0.095 \\ \frac{4}{3} \frac{\alpha_s}{\pi} \eta^*(\xi_{D_s^*}, r_c) &= -0.108 \end{aligned} \quad (16)$$

and for  $q = u$  :

$$\begin{aligned} \frac{4}{3} \frac{\alpha_s}{\pi} \eta(\xi_{D_s}, 0) &= -0.168 \\ \frac{4}{3} \frac{\alpha_s}{\pi} \eta^*(\xi_{D_s^*}, 0) &= -0.159 \quad . \end{aligned} \quad (17)$$

Using  $\tau_B = 1.6 \text{ ps}$  and  $|V_{cb}| = 0.04$ , one calculates, including the QCD corrections

$$Br(\bar{b} \rightarrow D_s^{(*)+} \bar{c}) \simeq 8.0\% \quad (18)$$

where  $Br(\bar{b} \rightarrow D_s^+ \bar{c}) \simeq 2.6\%$  and  $Br(\bar{b} \rightarrow D_s^{*+} \bar{c}) \simeq 5.4\%$  and with  $|V_{ub}|/|V_{cb}| = 0.08$

$$Br(\bar{b} \rightarrow D_s^{(*)+} \bar{u}) \simeq 6.8 \times 10^{-4} \quad (19)$$

where  $Br(\bar{b} \rightarrow D_s^+ \bar{u}) \simeq 2.3 \times 10^{-4}$  and  $Br(\bar{b} \rightarrow D_s^{*+} \bar{u}) \simeq 4.5 \times 10^{-4}$ .

At this stage, several points should be underlined:

- The sensitivity of the rate to the b quark mass goes as  $m_b^3$  instead of  $m_b^5$  in the case of the semileptonic decay.



- The sensitivity of the decay rate with respect to the mass  $m_q$  is negligible for the light quarks. It is not dramatic for the c quarks, in particular if  $m_b - m_c$  is known to a good accuracy (Eq. 8).
- The calculated overall branching fraction for  $Br(\bar{b} \rightarrow D_s^{(*)+}\bar{c}) \simeq 8.0\%$  is in agreement within  $1\sigma$  with the value measured [12] :

$$BR(B \rightarrow D_s^\pm X) = (10.0 \pm 2.5)\% \quad . \quad (20)$$

The observed agreement is encouraging as it shows that the very simple approach at the quark level accounts rather well for the data. Equivalently, one could extract  $|V_{cb}|$ . Using  $m_b = (5.0 \pm 0.20) \text{ GeV}/c^2$  and the relative error  $\sigma(f_{D_s^{(*)}})/f_{D_s^{(*)}} = 0.1$ , we find  $|V_{cb}| = 0.044 \pm 0.008$ .

- On the theoretical level it would be necessary to investigate these inclusive processes using the  $1/m_b$  expansion. On the one hand, one would need to estimate the next-to-leading non-perturbative corrections. On the other hand, a systematic analysis of these inclusive hadronic processes  $B \rightarrow D_s^\pm X$  could hopefully give independent information on those non-perturbative parameters of the heavy quark expansion such as  $\bar{\Lambda}$ ,  $\lambda_1$  and  $\lambda_2$  [13].

### 3 Measurement of $|V_{ub}|$ using $B\bar{B}$ pairs from $\Upsilon(4S)$ decays

In a similar way than for the measurement of  $|V_{cb}|$ , it should be possible to determine  $|V_{ub}|$  by selecting  $D_s$  mesons with momentum above the kinematical limit for  $B \rightarrow D_s^+ \bar{D}$ . The  $D_s$  momentum in the latter case is  $1.82 \text{ GeV}/c$  in the B rest frame. However, for B pair production at the  $\Upsilon(4S)$ , B mesons are generated with a

momentum of about 300 MeV/c and therefore the latter limit is of the order of 2.0 GeV/c as can be seen in figure 2. To extract  $|V_{ub}|$ , it is thus necessary to estimate the fraction of  $B \rightarrow D_s^+ X_u$  decays with  $p_{D_s} > 2.0$  GeV/c. We have computed the expected momentum spectrum of  $D_s^+$  produced via the spectator diagram in figure 1, taking into account the  $b$ -quark Fermi motion inside the  $B$  meson using the ACCMM model [14] at tree level, neglecting for the moment the radiative corrections. This spectrum is shown in figure 3. The striking feature of this distribution is that the average  $D_s$  momentum is above 2.0 GeV/c with about 75% of the  $D_s$  mesons above that limit. Obviously this fraction depends on the theoretical parameter and therefore we have varied  $p_F$  in the reasonable range ( $200 \text{ MeV}/c < p_F < 400 \text{ MeV}/c$ ) to evaluate the possible systematic uncertainties related to that parameter. Table 2 shows the sensitivity of the fraction of  $D_s$  for various cuts on  $p_{D_s}$ , assuming different values for  $p_F$  and the mass of the spectator quark. Should it be possible to measure the recoiling mass to the  $D_s^{(*)}$ , the value of the cut on  $p_{D_s}$  could be reduced, thus increasing the efficiency.

$m_u \ m_d(\text{MeV})$	$p_F \ (\text{MeV})$	$p_{D_s} > 2\text{GeV}/c$			$p_{D_s} > 2.05\text{GeV}/c$			$p_{D_s} > 2.1\text{GeV}/c$		
		$D_s$	$D_s^*$	all	$D_s$	$D_s^*$	all	$D_s$	$D_s^*$	all
150	300	76	36	50	65	27	39	53	19	30
10	200	89	50	63	82	39	54	71	29	43
10	300	82	44	57	74	34	48	63	25	38
10	400	74	38	51	65	30	42	55	22	33

Table 2: Efficiencies (in %) for a cut on the  $D_s$  momentum at 2, 2.05 and 2.1 GeV/c for four sets of values for the parameters of the ACCMM model.

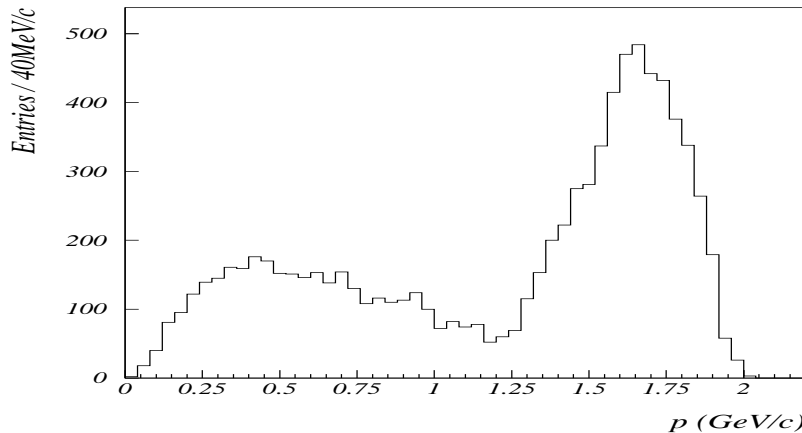


Figure 2: Momentum spectrum for  $D_s^+$  mesons produced from the reaction  $\bar{b} \rightarrow D_s^+ \bar{c}$  (i.e. upper vertex). Decays with a  $D^{**}$  meson from the lower vertex have not been included in this plot. These decays tend to fill the slight deep at 1.25 GeV/c but do not affect the end of the spectrum.

## 4 Backgrounds with tagged events

Let us now discuss in more detail, the issues raised using this method. The main questions are:

- Since we are not dealing with free quarks but B mesons, to which extent does the factorization for the decay  $B \rightarrow D_s^+ X$  hold ?
- Are there other means to produce  $D_s$  mesons ?

The former question will not be addressed here beyond repeating that factorization seems to hold for the color allowed decays when confronting the data. Furthermore, we have computed [9]  $O(\alpha_s)$  corrections to the naive formula (8).

Other production sources of  $D_s$  are shown in figure 4. In the following, we discuss these various  $D_s$  production mechanisms, evaluate their rate and propose means to reject the ones involving  $\bar{b} \rightarrow \bar{c}$  transitions or correct for the others. We should distinguish between the background that concerns tagged or untagged events. Let

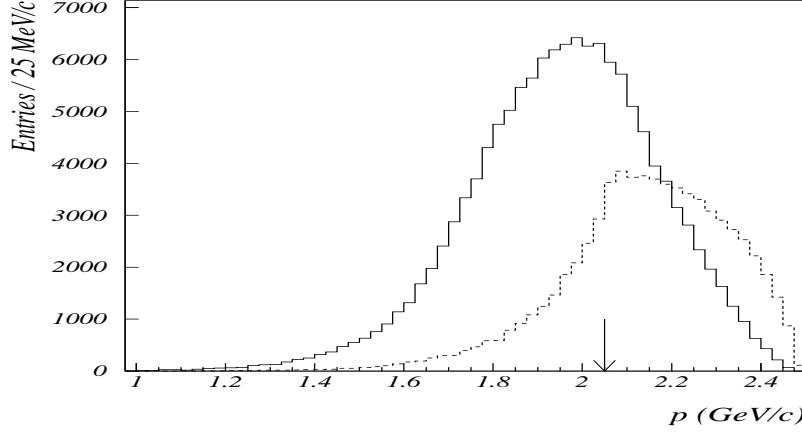


Figure 3: Expected momentum spectrum for  $D_s^+$  mesons produced from the reaction  $\bar{b} \rightarrow D_s^+ \bar{u}$  (i.e. upper vertex). The dashed line is for direct  $D_s^+$  while the solid line is for  $D_s^+$  mesons coming from direct  $D_s^{*+}$  decays.

us begin here with tagged events. In  $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ , assume that the  $\bar{B}$  is identified through its semileptonic decay. Then, the right sign  $D_s^+$  can be produced, besides the main mechanism of Fig. 1, by mechanisms of Figs. 4(a)-(d).

The  $c\bar{c}$  *continuum* background (Fig. 4a) has a large cross section,  $\sim 1.1nb$ . However, these events tend to have a jet-like structure and therefore can be rejected to a large extent by topological cuts. Furthermore, since a  $D_s$  meson has to be produced, the creation of a  $s\bar{s}$  pair is required, reducing the rate by about an order of magnitude. In addition, the momentum spectrum of the  $D_s$  meson produced in the continuum has a mean value larger than  $m_B/2$  reducing further this background by more than a factor 3. Finally, it is possible to subtract the remaining background by taking data just below the threshold for  $B\bar{B}$  production.

The *annihilation* diagram in Fig. 4b is obtained from the calculation of the inclusive rate  $B^+ \rightarrow c\bar{s}$  using:

$$J_{\mu q} = \bar{s}\gamma_\mu(1 - \gamma_5)c \quad (21)$$

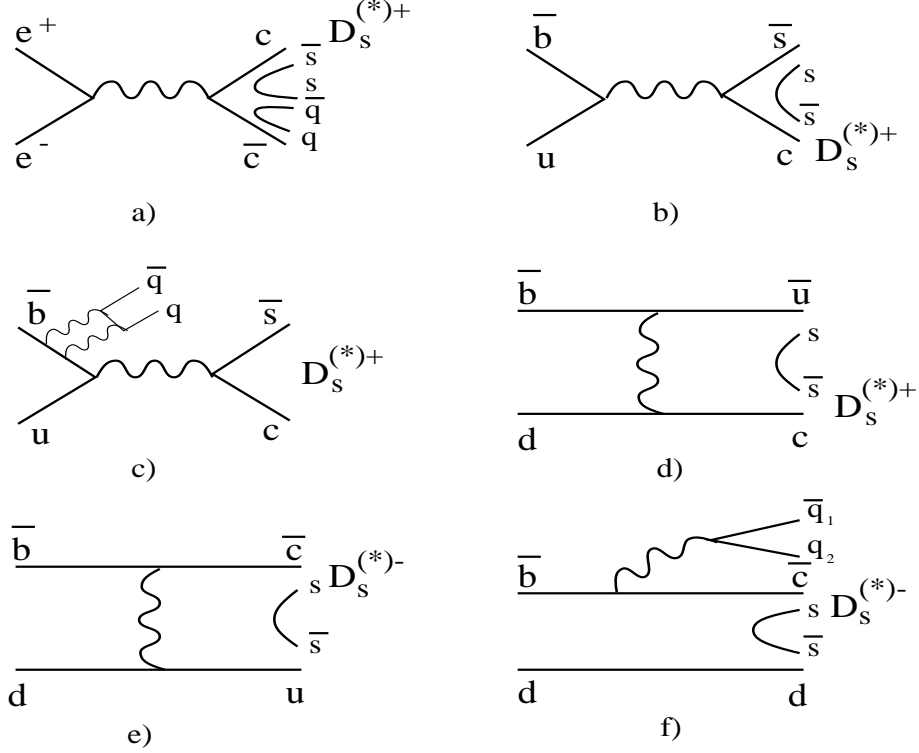


Figure 4: Diagrams leading to the production of  $D_s$  mesons.

$$\langle B^+ | A^\mu | 0 \rangle = -i f_B p_B^\mu \quad (22)$$

that gives [15]

$$\begin{aligned} \Gamma^{(0)}(B^+ \rightarrow c\bar{s}) &= \frac{N_c G_F^2}{8\pi} |V_{ub}^* V_{cs}|^2 f_B^2 m_B (m_c^2 + m_s^2) \left( 1 - \frac{(m_c^2 - m_s^2)^2}{m_B^2 (m_c^2 + m_s^2)} \right) \\ &\times \sqrt{1 - 2 \frac{(m_c^2 + m_s^2)^2}{m_B^2} + \frac{(m_c^2 - m_s^2)^2}{m_B^4}} a_1^2 \end{aligned} \quad (23)$$

Neglecting the  $s$  quark mass, one gets

$$\Gamma^{(0)}(B^+ \rightarrow c\bar{s}) \simeq \frac{N_c G_F^2}{8\pi} |V_{ub}^* V_{cs}|^2 f_B^2 m_B m_c^2 \left( 1 - \frac{m_c^2}{m_B^2} \right)^2 a_1^2 \quad (24)$$

Taking into account that one needs to create a  $s\bar{s}$  pair in order to obtain a  $D_s^{(*)}$  meson, one can assume

$$\Gamma^{(0)}(B^+ \rightarrow D_s^{(*)+} X) \leq \frac{1}{3} \Gamma^{(0)}(B^+ \rightarrow c\bar{s}) \quad (25)$$

Since  $m_B^2 f_B \cong m_D^2 f_D$  in the heavy quark limit the suppression factor of this mechanism relative to the spectator quark model (8) will be of the order or smaller than

$$\frac{N_c}{3} \left( \frac{m_c}{m_b} \right)^3 \sim 3 \times 10^{-2} \quad (26)$$

This branching fraction is small compared to the one deduced from Fig. 1 and would represent a small correction. The contribution from the diagram in Fig. 4c requiring the coupling via 2 gluons is expected to be much smaller and can be neglected.

The *exchange* diagram shown in Fig. 4d is evaluated in the same way than the annihilation one using:

$$J_{\mu q} = \overline{q_2} \gamma_\mu (1 - \gamma_5) q_1 \quad (27)$$

$$\langle B^0 | A^\mu | 0 \rangle = -i f_B p_B^\mu \quad (28)$$

and replacing  $a_1$  with  $a_2 = c_2 + c_1/N_c$  (color-suppressed process)

$$\Gamma^{(0)}(B^o \rightarrow q_1 \overline{q_2}) \simeq \frac{N_c G_F^2}{8\pi} |V_{q_2 b}^* V_{q_1 d}|^2 f_B^2 m_B m_c^2 \left( 1 - \frac{m_c^2}{m_B^2} \right)^2 a_2^2 \quad (29)$$

where  $q_1 \overline{q_2}$  can either be  $c\overline{u}$  or  $u\overline{c}$ . One should keep in mind that in this case the factorization Ansatz is on much weaker ground. Obviously, the case with  $q_1 = c$  and  $\overline{q_2} = \overline{u}$  is suppressed since the CKM factors are  $|V_{ub}^* V_{cd}|$ . This means that this mechanism in the case of tagging is Cabibbo suppressed and color suppressed relatively to the main mechanism of Fig. 1. Comparing (8) to (29), the reduction factor is of the order

$$\tan^2 \theta_c \frac{N_c}{3} \left( \frac{m_c}{m_b} \right)^3 \left( \frac{a_2}{a_1} \right)^2 \lesssim 10^{-4} \quad (30)$$

and we can safely neglect this mechanism. The conclusion is that, if there is tagging, the mechanisms that can compete with the interesting process of Fig. 1 either can

be discarded by kinematical cuts or are smaller by a factor of the order  $10^{-2}$ . The method seems therefore safe if there is tagging.

## 5 Backgrounds with untagged events

If in  $e^+e^- \rightarrow B\bar{B}$  we assume no tagging, besides the additional mechanisms of Figs. 4 a-d, we can have also the processes 4 e-f, that lead to a wrong sign  $D_s$ . First, one must remark that the *continuum* background can also lead to a wrong sign  $D_s$  (the lower  $D$  in the diagram 4a), but we know that one can dispose off of these events by topological cuts. Also, the *exchange* process 4e, that corresponds to replacing in 4d  $\bar{u} \rightarrow \bar{c}$  and  $c \rightarrow u$ , can lead to a wrong sign  $D_s$ . Unlike the case with  $q_1 = u$  and  $\bar{q}_2 = \bar{c}$  this process is in principle enhanced because the CKM factors are  $|V_{cb}^*V_{ud}|$ . Similarly a  $s\bar{s}$  pair is required to get a  $D_s^{(*)+}$  meson and therefore with  $|a_2| = 0.2$  one obtains a naive suppression factor smaller than

$$\frac{1}{3} \left| \frac{V_{cb}}{V_{ub}} \right|^2 N_c \left( \frac{m_c}{m_b} \right)^3 \left( \frac{a_2}{a_1} \right)^2 \cong 0.20 \quad . \quad (31)$$

Although this source is only present for neutral B decays and is smaller than the spectator diagram in Fig 1, the corresponding branching fraction could be non negligible, and its possible suppression relies on dynamical assumptions that are not very reliable. This branching fraction and (30) as well may further be enhanced by the emission of gluon from the initial light quark. In this case [16,17], the most important changes relative to (29) are the absence of the  $m_c^2/m_B^2$  dependence due to helicity and the presence of the factor  $f_B^2/m_d^2$  instead of  $f_B^2/m_B^2$  due to the gluon radiation from the initial light quark. Therefore, gluonic emission may enhance the rate of the exchange diagram by one order of magnitude if one uses  $m_d = 300 \text{ Mev}/c^2$  since the d quark must be interpreted as a constituent quark in this process. However,

as pointed out in [18], the presence of the infrared sensitive parameter  $1/m_d^2$  makes problematic a rigorous perturbative estimation of this contribution. Furthermore, in the full inclusive decay, according to Heavy Quark Theory, this type of contributions should be suppressed by a factor  $1/m_b^3$  relative to the main spectator diagram. On the other hand, present limits (for example  $Br(B^0 \rightarrow D_s^- K^+) < 2.4 \times 10^{-4}$  [19]) tend to disfavor a large enhancement. The same conclusion can be reached using D lifetime measurements [20]. It is nevertheless important to find a way to either measure it or to eliminate it. One possibility could be to observe some of these final states, for example  $D_s^{(*)-} K^{(*)+}$  and evaluate their contribution.

The CLEO collaboration has measured the rate  $\bar{b} \rightarrow D_s^- X$  [21] due to diagrams 4e-4f, although other sources exist (see next subsection). The total rate was found to be  $(2.1 \pm 1.0) \%$ . However the momentum spectrum of those  $D_s$  is expected to be rather soft with less than 0.5 % of those having a momentum greater than 2.0 GeV/c. This leads to an effective branching fraction  $Br(B \rightarrow D_s^- X [p_{D_s} > 2.0 \text{ GeV/c}]) < 1.5 \times 10^{-4}$  at 90% .

It is also possible to produce  $D_s^{(*)}$  mesons of the wrong sign in *multibody B decays* such as the one shown in Fig. 4f. The decay rate of this type of modes is potentially large. However, one should note several important points.

- The production of  $D^{**}$  with orbital excitation  $L=1$  would not lead to  $D_s^{(*)}$  as this meson needs to be accompanied by a kaon and the total mass  $D_s^{(*)} K$  is larger than the  $D^{**}$  mass.
- In the case of non resonant  $D_s^{(*)} K$  production from the lower vertex, the energy is shared between the final 3 or more particles and therefore the momentum



spectrum of the  $D_s^{(*)}$  is softer and barely reaches the range where  $B \rightarrow D_s^{(*)+} X_u$  is expected. As discussed in the above subsection, CLEO measurements indicate that this type of decay should not be a problem.

The CLEO measurement mentioned in the previous section shows that this background should not be large.

## 6 Feasibility study using the Babar detector

The feasibility of this new method for measuring  $|V_{ub}|$  has been verified for the Babar detector at the SLAC B factory PEP-II.

We have used the full detector simulation and the reconstruction program [22] to generate 5000 events with the following decay of one B meson  $B \rightarrow D^{(*)} D_s^{(*)}$ , where  $D_s$  decays to the  $\phi\pi$  final state, and  $\phi \rightarrow K^+ K^-$ . This gives a  $D_s$  spectrum peaked at  $1.7 \text{ GeV}/c$  in the center of mass system, therefore only slightly below the expected signal of  $D_s$  coming from  $V_{ub}$  transitions. Generic  $B\bar{B}$  decays were used to measure the background level.

We have studied two crucial points for this analysis :

- the reconstruction efficiency for the  $D_s$ ,
- the momentum resolution.

The analysis to isolate the  $D_s$  signal proceeds as following :  $K^\pm$  are identified using the combined information coming from the Silicon Vertex Tracker, the Drift Chamber and the DIRC detector and then selected if their invariant mass is in the  $1020 \pm 10 \text{ MeV}/c^2$  interval. A third track, assumed to be a pion, is then selected. A cut on  $\cos\psi$ ,  $|\cos\psi| > 0.4$ , where  $\psi$  is the angle between one Kaon and the  $D_s$

momentum in the  $\phi$  rest frame, is then applied.

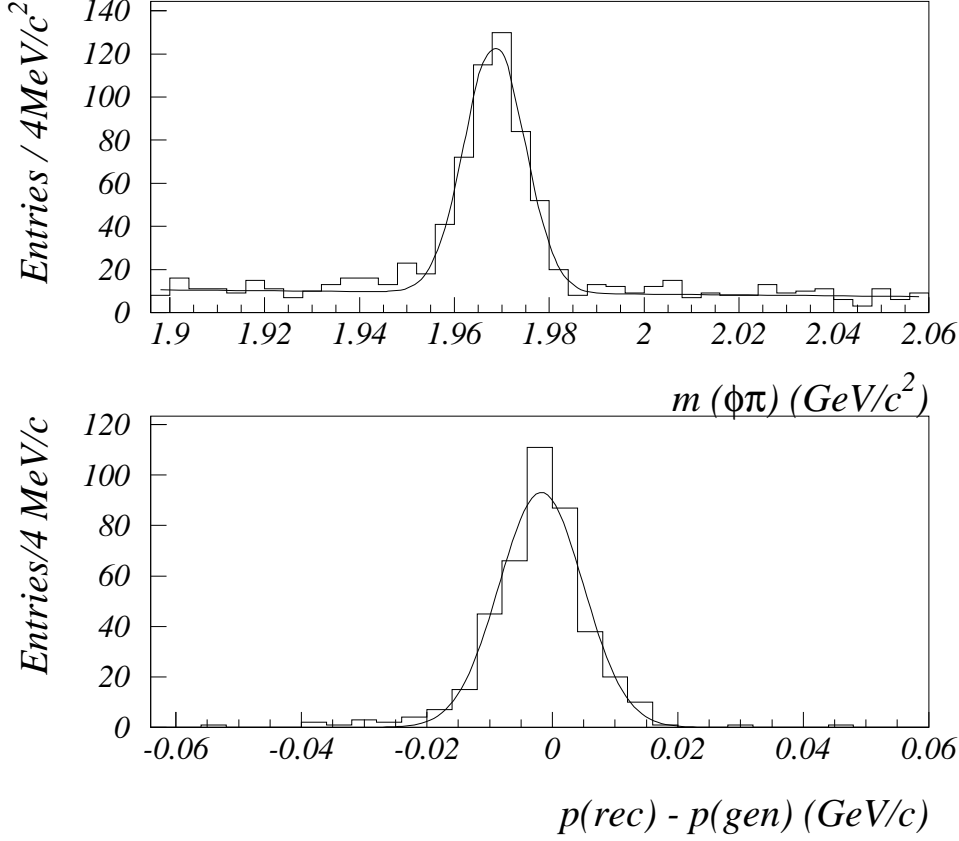


Figure 5: Mass and momentum resolution for  $D_s$  mesons using the full BaBar detector simulation and reconstruction programs.

The resolution on the  $D_s$  mass is  $6.4 \pm 0.3 \text{ MeV}/c^2$  (Fig. 5a).

The reconstruction efficiency is  $39 \pm 2 \%$  and the momentum resolution is  $6.7 \pm 0.3 \text{ MeV}/c$  (Fig. 5b). The latter result insures that there will be no leaking from the lower to higher momenta. This excellent resolution is due to the fact that we implicitly reject mismeasured tracks : these will not give a  $D_s$  candidate with the right invariant mass.

Using these results, we can compute the number of reconstructed signal events

that we expect. We have for untagged events

$$n_{rec} = 2 n_{B\bar{B}} Br(B \rightarrow D_s^{(*)} X_u) Br(D_s^+ \rightarrow \phi \pi^+) Br(\phi \rightarrow K^+ K^-) \epsilon_{rec} = 143$$

per  $30 fb^{-1}$ , the nominal integrated luminosity for one year of data taking at Babar.

We took  $Br(B \rightarrow D_s^{(*)} X_u) = 6.8 \cdot 10^{-4}$  from Eq. 19,  $Br(D_s^+ \rightarrow \phi \pi^+) = 3.5\%$ , and  $\epsilon_{rec}$  takes into account also the cut on  $D_s$  momentum at  $2.05 GeV/c$ . Therefore we can conclude that the number of reconstructed events will be sufficient to measure  $|V_{ub}|$  with a good statistical precision. This number can be improved by reconstructing the  $D_s$  meson in other modes.

As we have pointed out above, there are unwanted sources of  $D_s$  beyond the kinematical limit for  $B \rightarrow D_s^+ X_c$ , and it is suitable to be able to reject them experimentally. As we have emphasized, one way to do this is to tag the flavor of the recoil B meson in the event, for instance by considering its semileptonic decay. The correlation between the sign of the lepton and the sign of the  $D_s$  meson is opposite for  $B \rightarrow D_s^+ X_u$  and for the transitions due to the exchange diagrams of figure 4-e (as well as for multibody B decays like figure 4-f).

This method has already been used by other experiments like CLEO and Argus to study the lepton spectrum in the B semileptonic decays. A cut on the angle between the lepton and the  $D_s$  meson allows to reject the pairs due to a  $D_s$  and a lepton from the same B meson. The only major problem of this method is the further reduction of the selected sample it implies, which should be no larger than 5-10% of the number of reconstructed events estimated above. Therefore this is a possibility which is open but it would probably require a big experimental effort to reconstruct the largest possible fraction of  $D_s$  mesons to be really viable.

## 7 Conclusion

In conclusion, we have shown that the process  $B \rightarrow D_s^+ X_u$  can allow the determination of the CKM matrix element  $|V_{ub}|$  in  $e^+e^-$  collisions at the  $\Upsilon(4S)$ , as in the BaBar experiment. If there is tagging of one  $B$  meson, the prospects are very good since the backgrounds to the main spectator model mechanism, whose spectrum would allow the determination of  $|V_{ub}|$ , are either suppressed by a factor of the order of  $10^{-2}$ , or can be disposed off by kinematical cuts. However, the number of events is drastically reduced by tagging.

If tagging is not assumed, other mechanisms can give a large background, but the method could still work if theoretical and experimental studies of these additional processes leading to a wrong sign  $D_s$  are performed in the future. It should be noted that these wrong sign backgrounds (figures 4e and 4f) are Cabibbo enhanced but suppressed by color and other dynamical effects (Section 5). In contrast, in semileptonic decays, even when the hadronic background is studied [2], misidentified direct  $b \rightarrow c$  decays are Cabibbo enhanced and difficult to exclude kinematically because of the neutrino. Admittedly, the semileptonic method has the advantage of statistics.

We are aware that our study is a preliminary survey of the possibility of measuring  $|V_{ub}|$  with a new method. Work remains to be done. For the elementary processes  $\bar{b} \rightarrow D_s^{(*)+} \bar{q}$  ( $\bar{q} = \bar{u}, \bar{c}$ ), one would need to compute the spectrum taking into account the radiative corrections and comparison with the spectrum  $B \rightarrow D_s^\pm X$  needs to be done as a check. Up to now, only the integrated corrected rate has been computed [9]. On the other hand, theoretical or phenomenological work needs to be done to further

constrain the sources of background and of hadronic uncertainties in the case of tagged and also, hopefully, although more difficult, for untagged events.

We insist that the method proposed here, having very different systematic errors than the semileptonic one, would provide an irreplaceable check of  $|V_{ub}|$ .

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